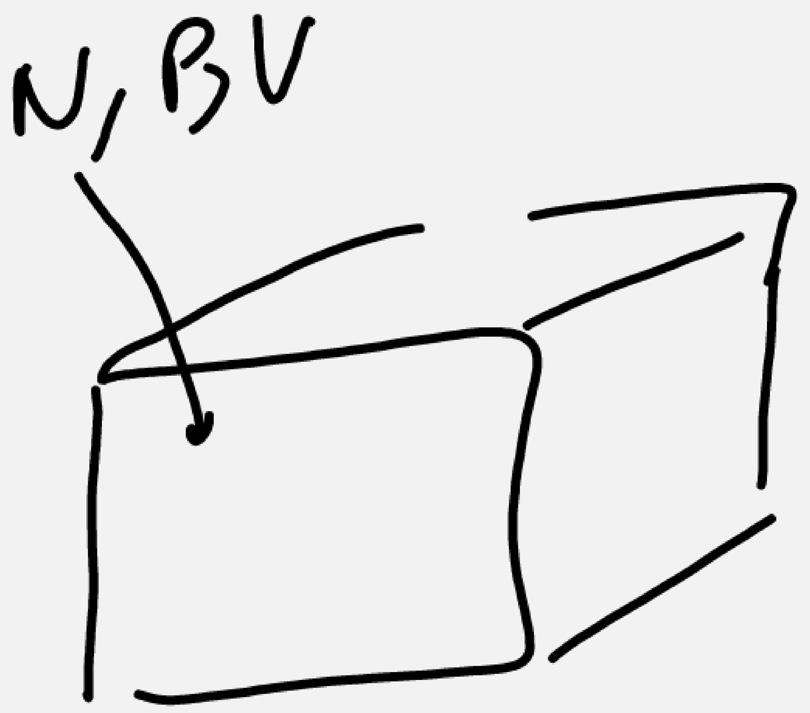


$$PV = N \times \frac{1}{3} m\langle v^2\rangle \times \frac{2}{2} = N \frac{2}{3}\langle E_c\rangle$$

XP : $PV = N k_B T$

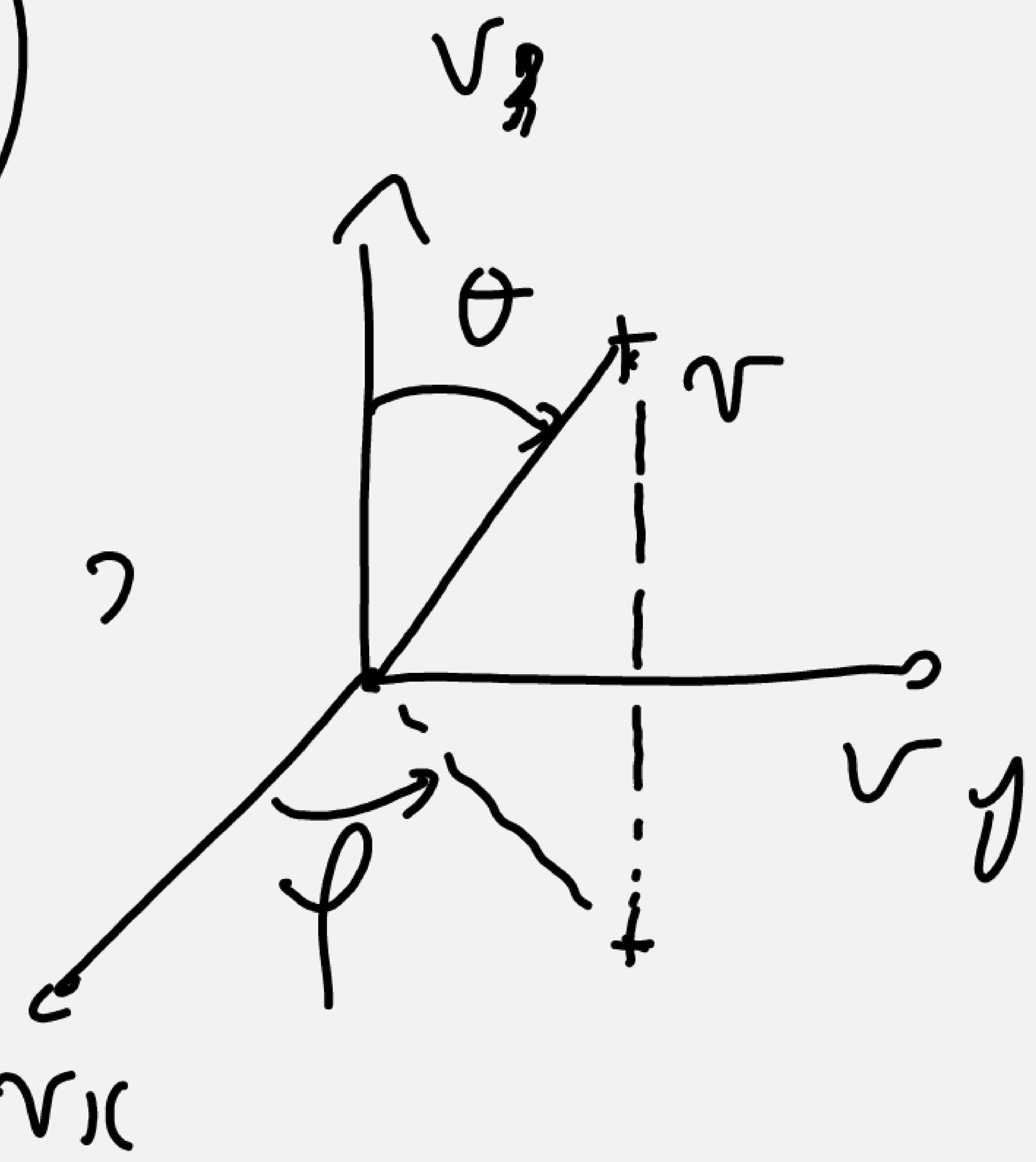
$= nRT$

$$\langle E_c\rangle = \frac{3}{2} k_B T$$

E tot : $\begin{pmatrix} v_x, v_y, v_z \\ x, y, z \end{pmatrix} \longleftrightarrow$ Energie

$$\boxed{P(\text{état}) = C \cdot \exp\left(-\frac{E_{\text{état}}}{k_B T}\right)}$$

exemple : $\rho(v_x, v_y, v_z) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m v^2}{2 k_B T}\right)$



$(v_x, v_y, v_z) \longleftrightarrow (v, \theta, \varphi)$

Rappels :

$$I_0 = \int_0^{+\infty} e^{-at^2}\,dt = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

ou $\left(\boxed{\int_{-\infty}^{+\infty} e^{-at^2}\,dt = \sqrt{\frac{\pi}{a}}}\right)$ INTEGRALE D'UNE GAUSSIENNE

Loi normale :

$$\mathcal{N}(m, \sigma^2) : \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \longrightarrow \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx = 1$$

ici $a = \frac{1}{2\sigma^2}$

---

on veut

$$\int_{-\infty}^{+\infty} \sqrt{\frac{m}{2\pi k T}} \exp\left(-\frac{mt^2}{2kT}\right) dt \qquad \sigma^2 = \frac{k_b T}{m}$$

$$= \sqrt{\frac{m}{2\pi k_b T}} \times \sqrt{\frac{2\pi k_b T}{m}} = 1$$

$$p(v_x, v_y, v_z) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m v^2}{2 k_B T}\right)$$

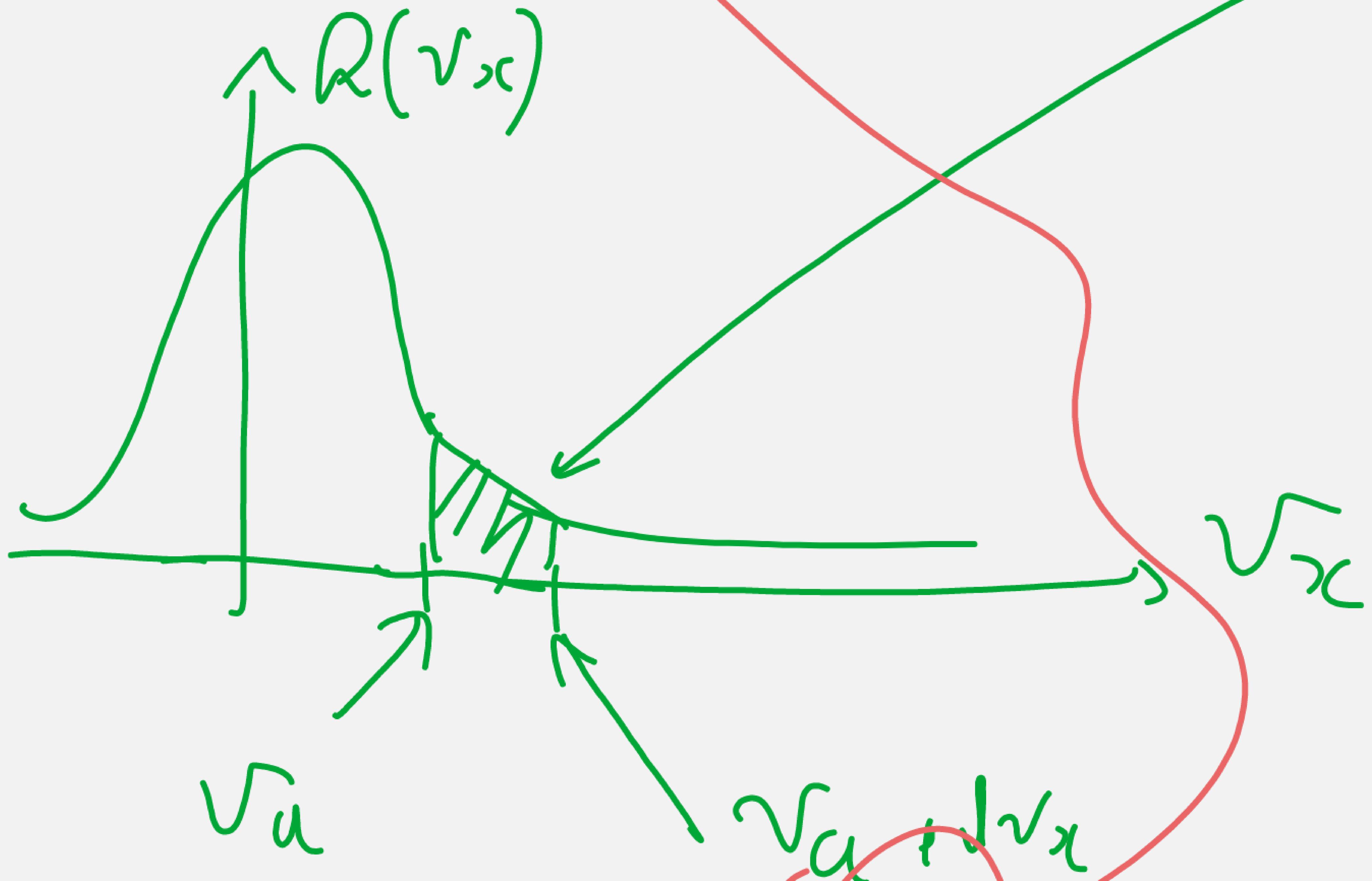
$$\underbrace{\left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{m v_x^2}{2 k_B T}\right)}_{R(v_x)} \underbrace{\sqrt{\frac{m}{2\pi k_B T}}\ \exp\left(-\frac{m v_y^2}{2 k_B T}\right) dv_y}_{R(v_y) \; = 1} \underbrace{\left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{m v_z^2}{2 k_B T}\right) dv_z}_{R(v_z) \; = 1}$$

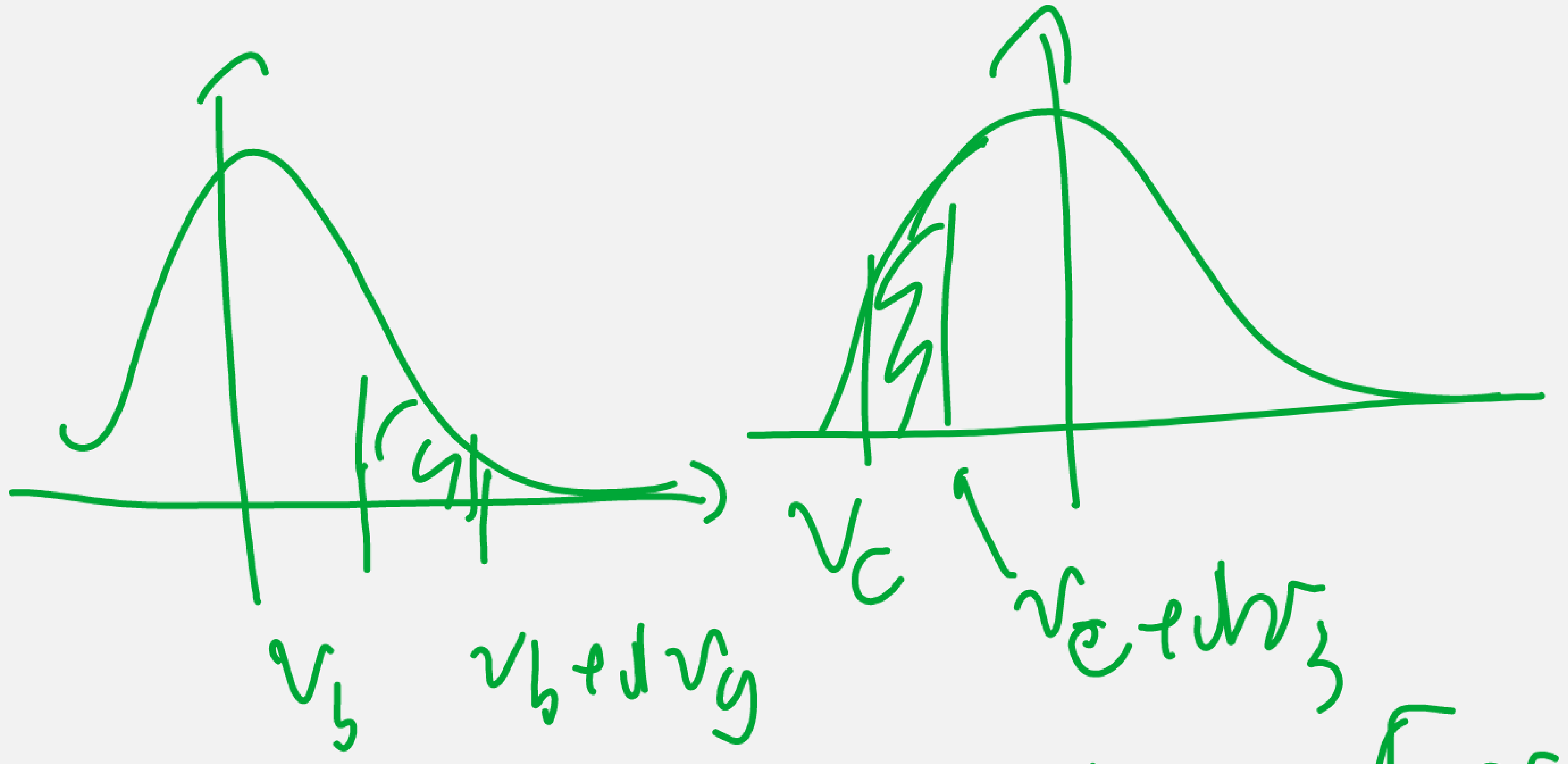
$$\rho(v_x) = \int_{v_y}\int_{v_z} p(v_x, v_y, v_z)\, dv_y\, dv_z = R(v_x)$$

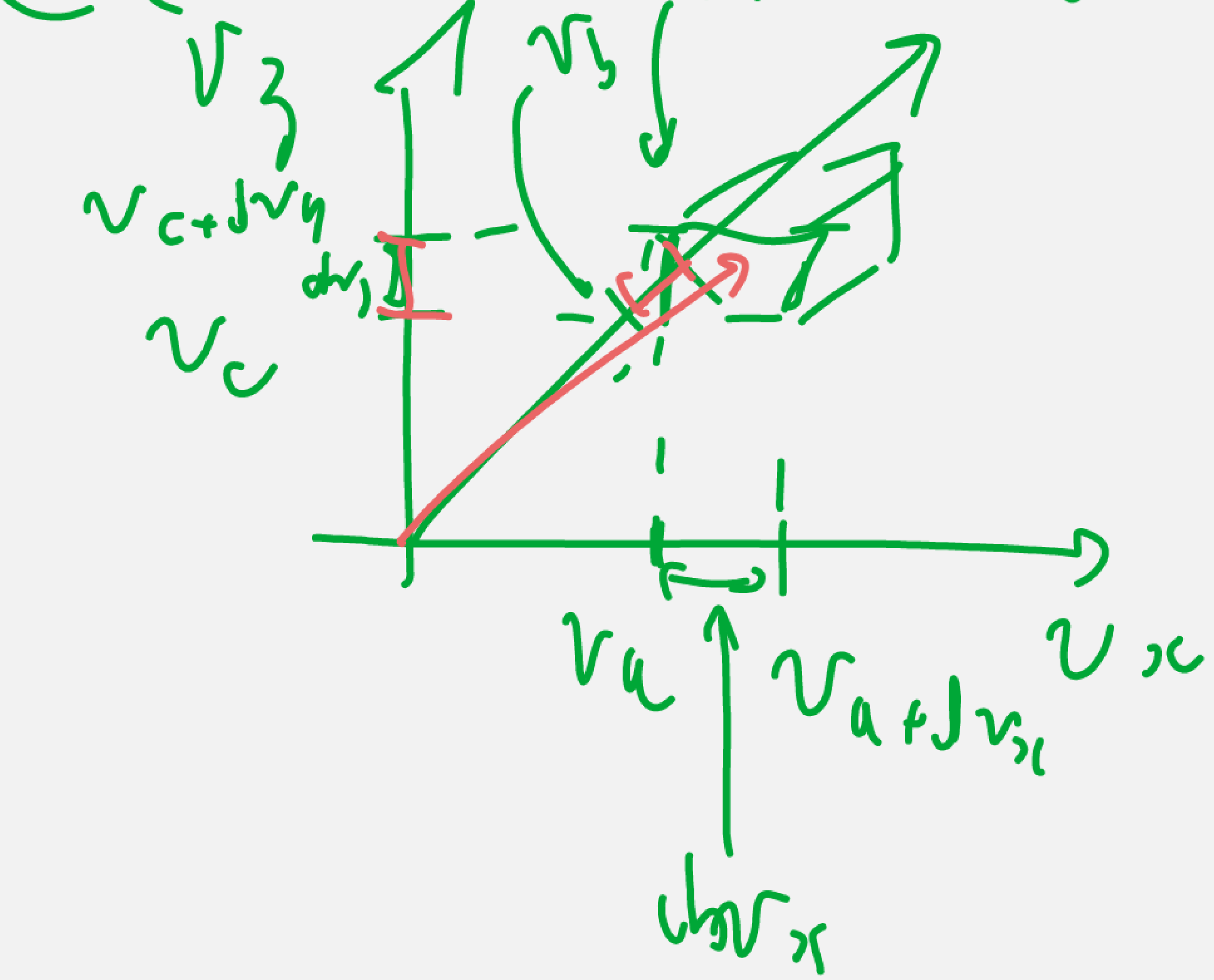
$$R(t) = \left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{m t^2}{2 k_B T}\right)$$

$$\boxed{\int_{-\infty}^{+\infty} R(t)\, dt = ?}$$

$$f(v_a, v_b, v_c) = \underbrace{R(v_a)\,dv_x} \times R(v_b)\,dv_y \times R(v_c)\,dv_z$$







$R(v_a)\,dv_x$: Proba pour v_x d'être dans $[v_a, v_a + dv_x]$

$R(v_b)\,dv_y$: " " v_y " " $[v_b, v_b + dv_y]$

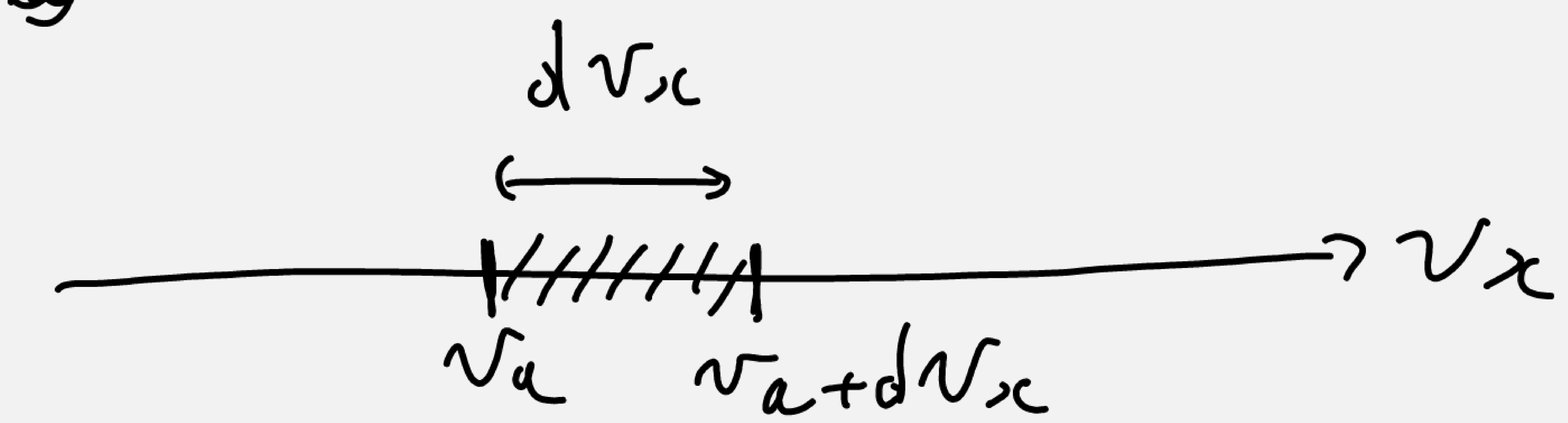
$R(v_c)\,dv_z$: Proba " v_z " " $[v_c, v_c + dv_z]$

$$\rho(v_x, v_y, v_z) = \underbrace{R(v_x)} \cdot R(v_y) \cdot R(v_z)$$

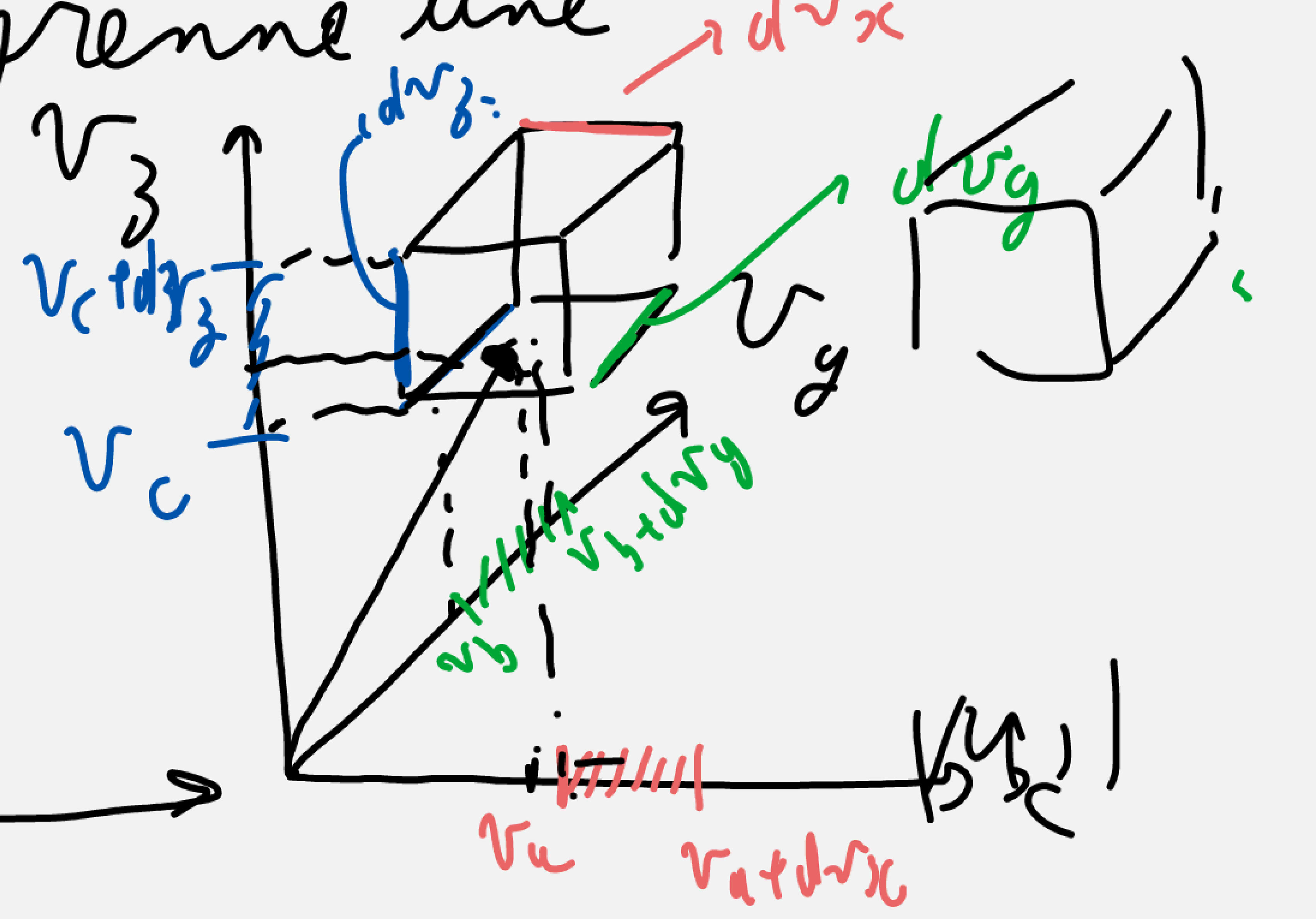
$$R(v_i) = \left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{m v_i^2}{2 k_B T}\right)$$

$i = x, y, z$

$R(v_a)$: densité de probabilité que la composante v_x de la vitesse en v_a



$R(v_a)\, dv_x$: Proba que la composante v_x de la vitesse prenne une valeur dans $[v_a, v_a + dv_x]$



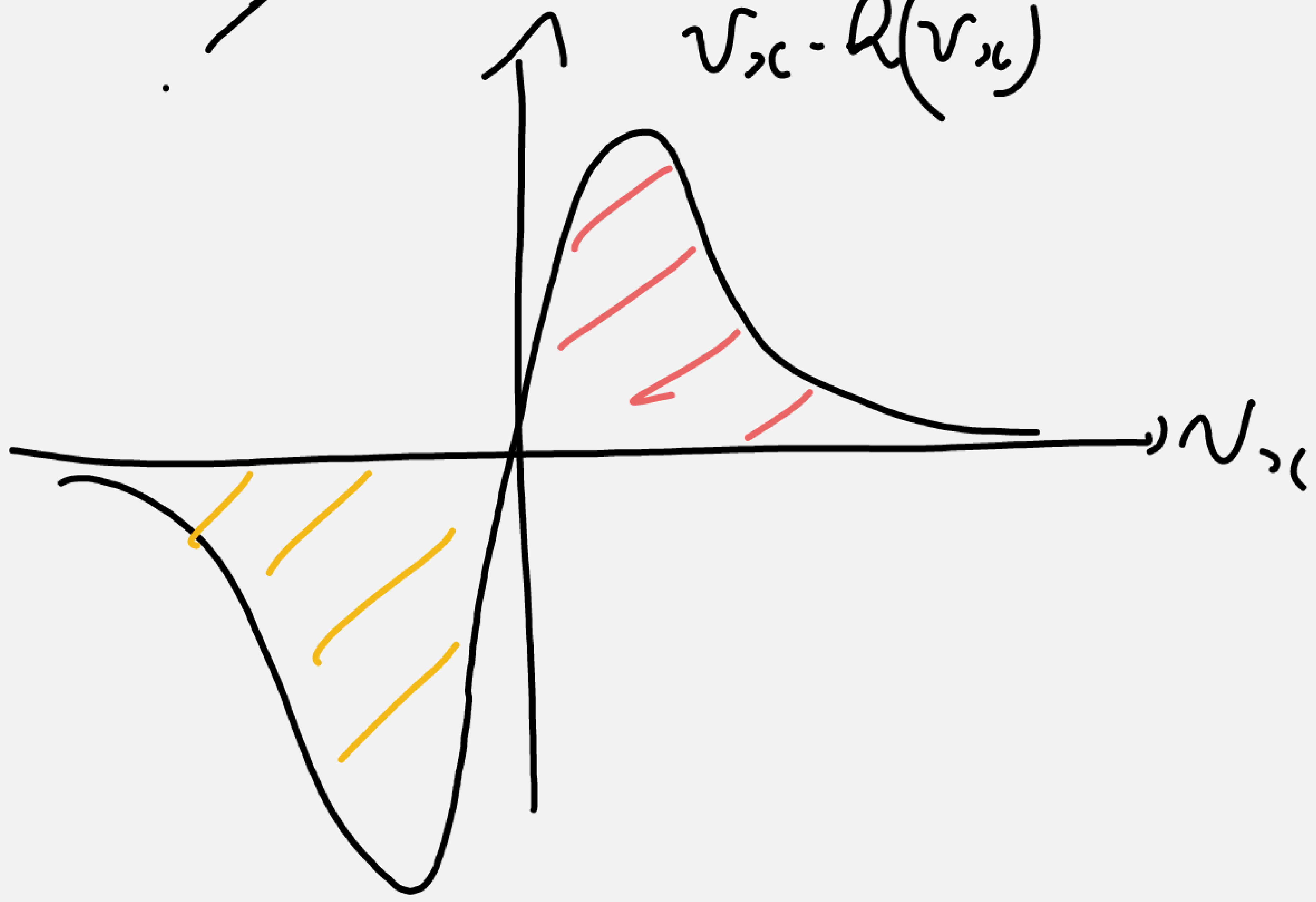
$$\underbrace{\rho(v_a, v_b, v_c)}\, \underbrace{dv_x\, dv_y\, dv_z}$$

$$R(v_x) = \left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{m v_x^2}{2 k_B T}\right)$$

$$\langle E_{cx} \rangle = \frac{1}{2} m \langle v_x^2 \rangle = \frac{1}{2} k_B T$$

$$E(E_{cx}) = \frac{1}{2} m \, E(v_x^2)$$

$$E(v_x) = \int_{-\infty}^{+\infty} \underbrace{\overset{\text{impaire}}{v_x} \; \overset{\text{Paire}}{R(v_x)} \, dv_x}_{\text{Impaire}} = 0$$



$P_x(k) = \left(\frac{1}{6}\right) \ \forall k.$

v_x

Espérance de X:

k	k^2
1	1
2	4
3	9
4	16
5	25
6	36

$P(k^2) = P(k)$

v.a. discrète

$$E(X) = \langle X\rangle = \sum_{\boxed{k\in E}} k \times P_x(k)$$

$E = \{1, 2, \ldots, 6\}$

$-2 \ -1 \ 0 \ 1 \ 2$

$P(4) = \frac{2}{5}$, $P(1) = \frac{2}{5}$, $P(0) = \frac{1}{5}$

v.a. continue

$$\langle X\rangle = \int_E x \times \rho_x(x)dx$$

$$\langle v_x\rangle = \int v_x \, R(v_x)\, dv_x$$

$\boxed{E = \mathbb{R}}$

v.a.

$k \longleftrightarrow v_x$

$$E(X^2) = \sum_{k\in\{1\ldots 6\}} k^2 P_x(k)$$

$$E(v_x^2) = \int_{E=\mathbb{R}} v_x^2 \, R(v_x)\, dv_x$$

$\langle E_c \rangle = \langle \frac{1}{2} m v_x^2 \rangle = \frac{1}{2} m \langle v_x^2 \rangle$

Moments (n ∈ N):

v.a. discrète

$$m_n = \sum_{k \in E} k^n \times P_x(k)$$

v.a. continue

$$m_n = \int_E x^n \times \rho_x(x) dx$$

Moyenne d'une fonction f

v.a. discrète

$$\langle f \rangle = \sum_{k \in E} f(k) \times P_x(k)$$

$E(\rho(k))$

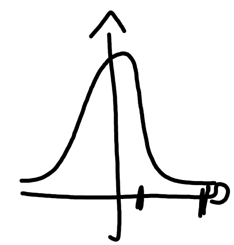
v.a. continue

$$\langle f \rangle = \int_E f(x) \times \rho_x(x) dx$$

A vos stylos !

$\rho_x(x)$ Normée ?
m espérance ?
σ^2 variance ?

$$\rho_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$$



$$I_n = \int_0^{\infty} t^n e^{-at^2} dt$$

$$I_{n+2} = \frac{n+1}{2a} I_n$$

$$I_0 = \frac{1}{2}\sqrt{\frac{\pi}{a}} \quad \text{et} \quad I_1 = \frac{1}{2a}$$

$$R(v_x) = \sqrt{\frac{m}{2\pi k_B T}} \exp\left(-\frac{m v_x^2}{2 k_B T}\right)$$

$g(v_x)$

$$E(g(v_x)) = \int_{-\infty}^{+\infty} \boxed{g(v_x)} \cdot R(v_x)\, dv_x$$

$$g(v_x) = \frac{1}{2} m v_x^2$$

$$E(E_{cx}) = E\left(\frac{1}{2} m v_x^2\right)$$

$$E(E_{cx}) = \left(\frac{m}{2\pi k_B T}\right)^{1/2} \times \frac{1}{2} m \int_{\cancel{-\infty}\, 0}^{+\infty} v_x^2 \exp\left(-\frac{m v_x^2}{2 k_B T}\right) dv_x \quad \times 2$$

Paire

$$E(E_{cx}) = \cancel{\left(\frac{m}{2\pi k_B T}\right)}^{1/2} \times \frac{1}{\cancel{2}} \cancel{m} \times \cancel{2} \times \frac{k_B T}{\cancel{m}} \times \frac{1}{2} \sqrt{\cancel{\frac{2\pi k_B T}{m}}} = \frac{k_B T}{2}$$

$t \leftrightarrow v_x$

$a \leftrightarrow \frac{m}{2 k_B T}$

Equi-Partition de l'energie.

$v_x = v \sin\theta \cos\varphi$

$v_y = v \sin\theta \sin\varphi$

$v_z = v \cos\theta$

$v \to dv + v$ $\qquad v \in [0, +\infty[$

$\theta \to d\theta + \theta$ $\qquad \theta \in [0, \pi]$

$\varphi \to d\varphi + \varphi$ $\qquad \varphi \in [0, 2\pi]$

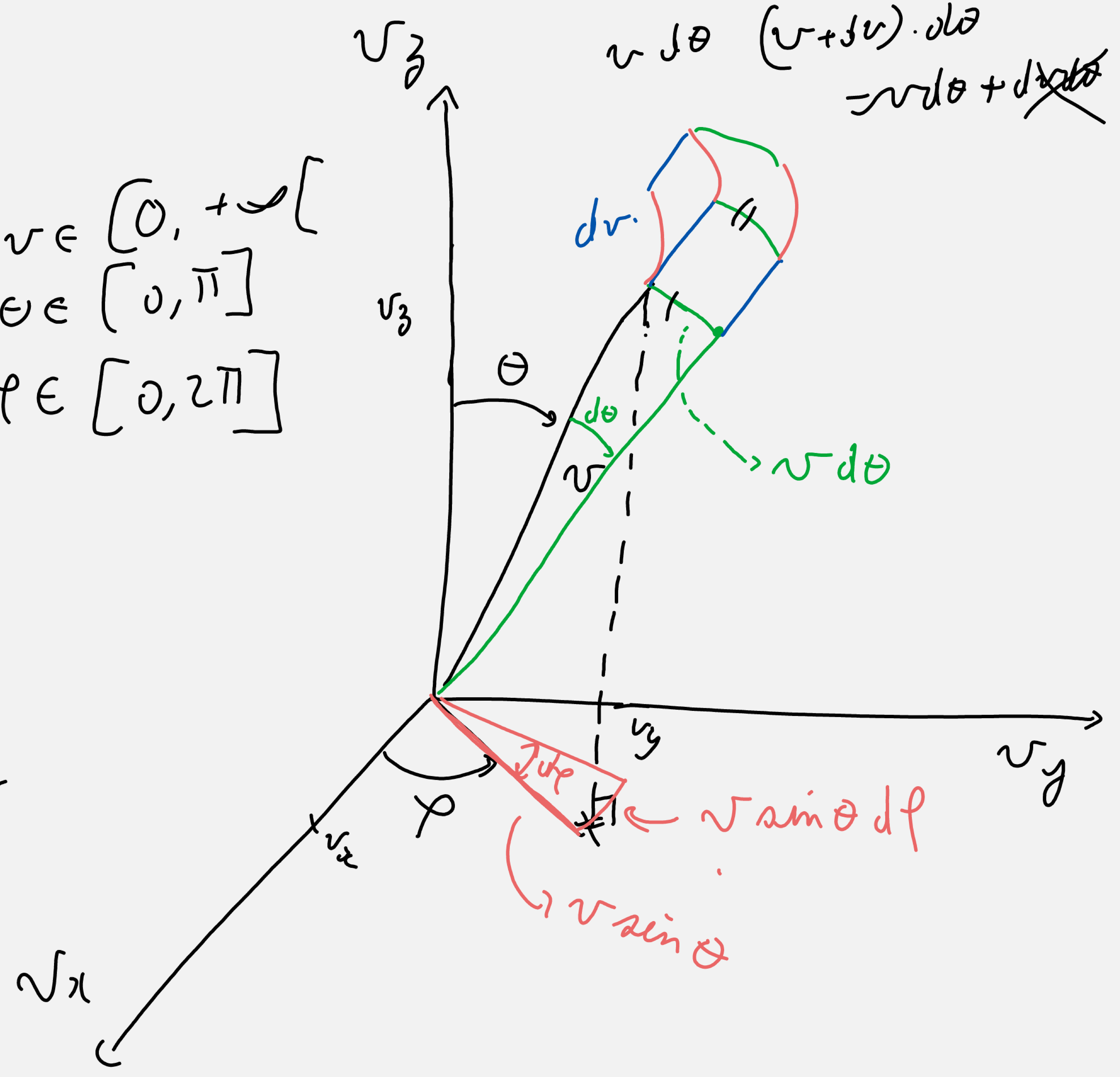
θ : Co-latitude

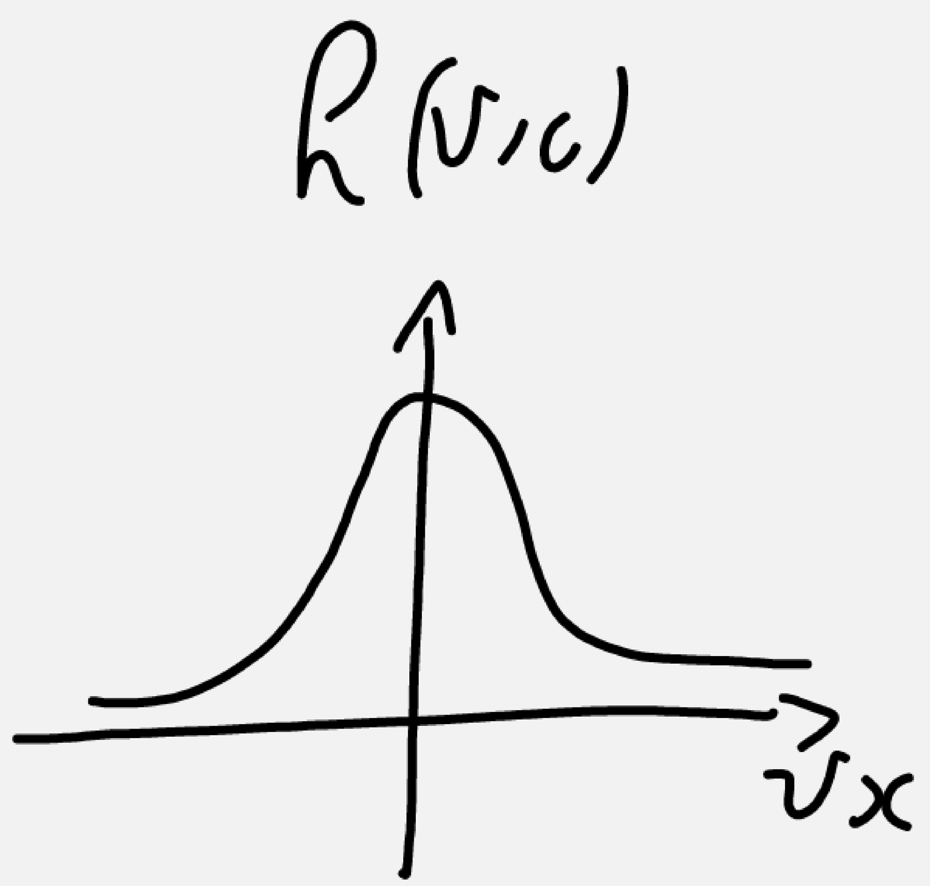
φ : longitude

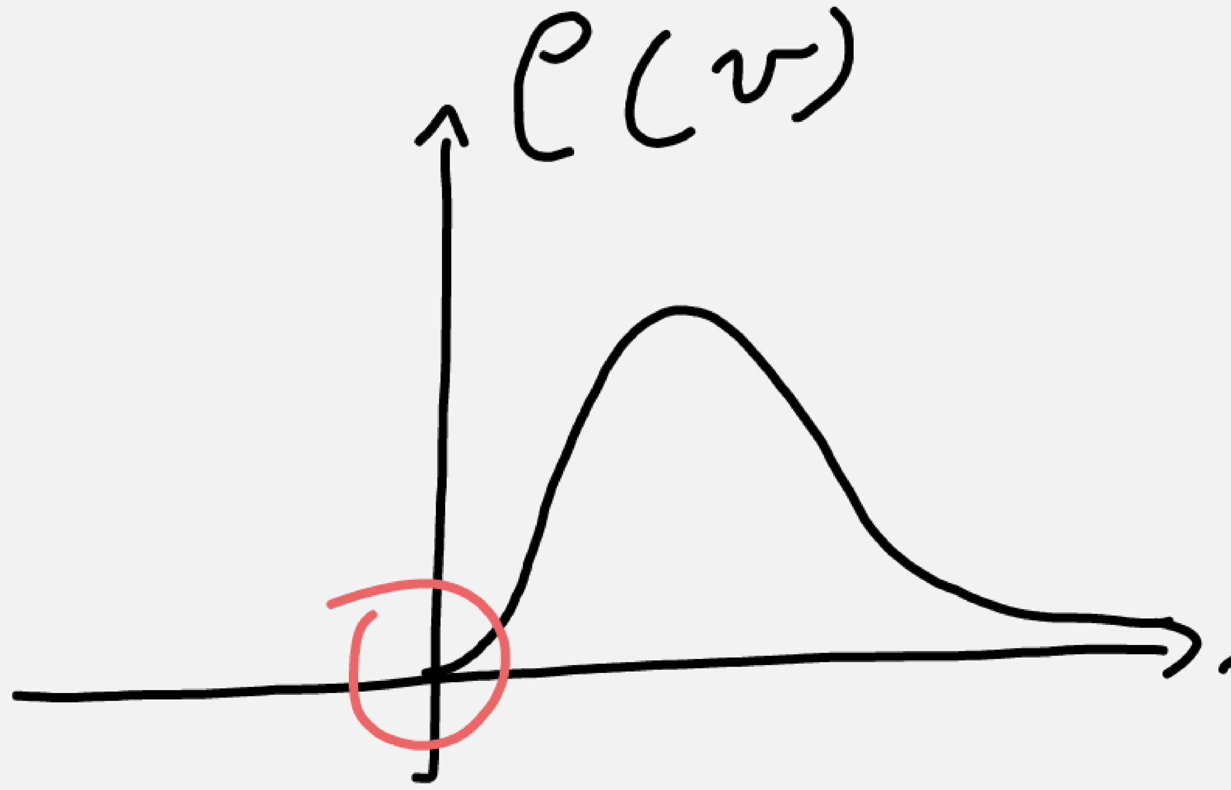
Élément de volume :

$$dv \cdot v\,d\theta \cdot v \sin\theta\, d\varphi$$

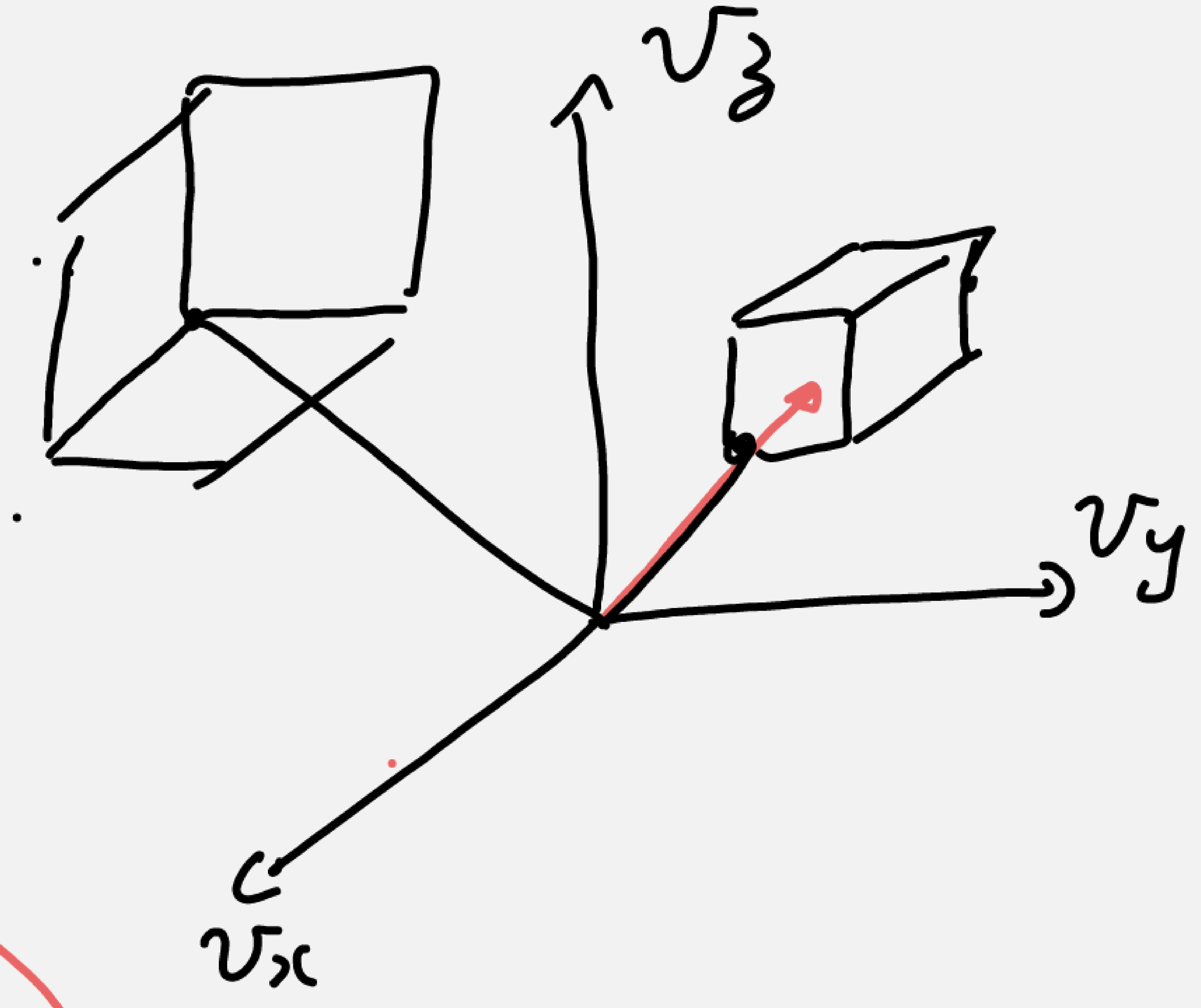
$$dv_x\,dv_y\,dv_z \longleftrightarrow v^2 \sin\theta\, d\theta\, d\varphi\, dv$$

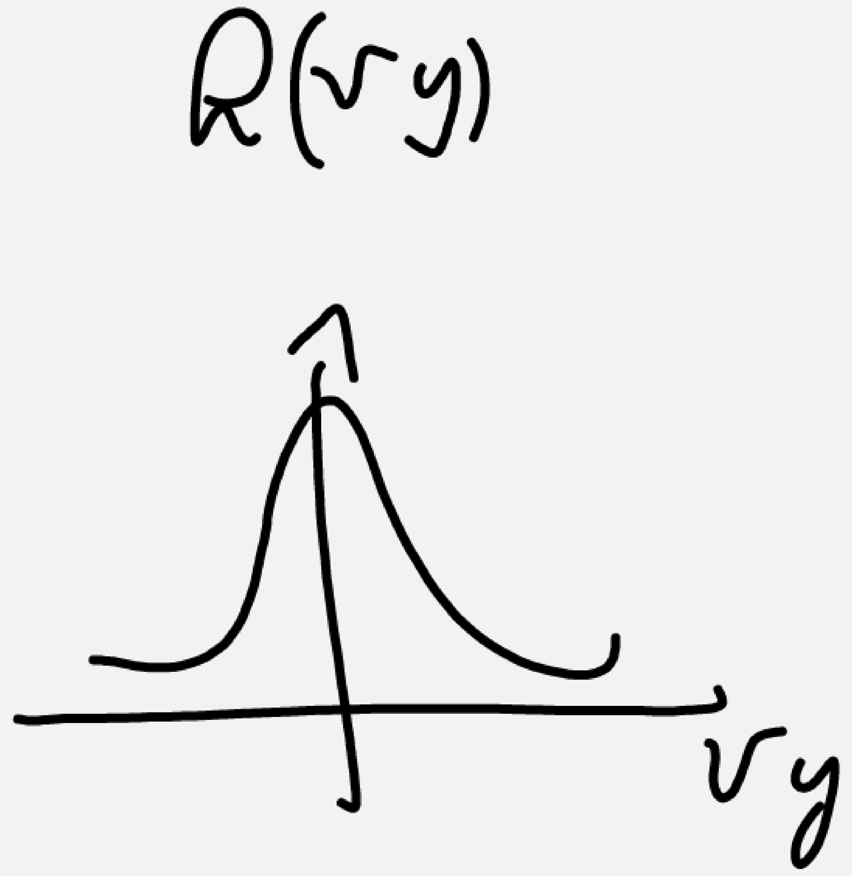






$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$



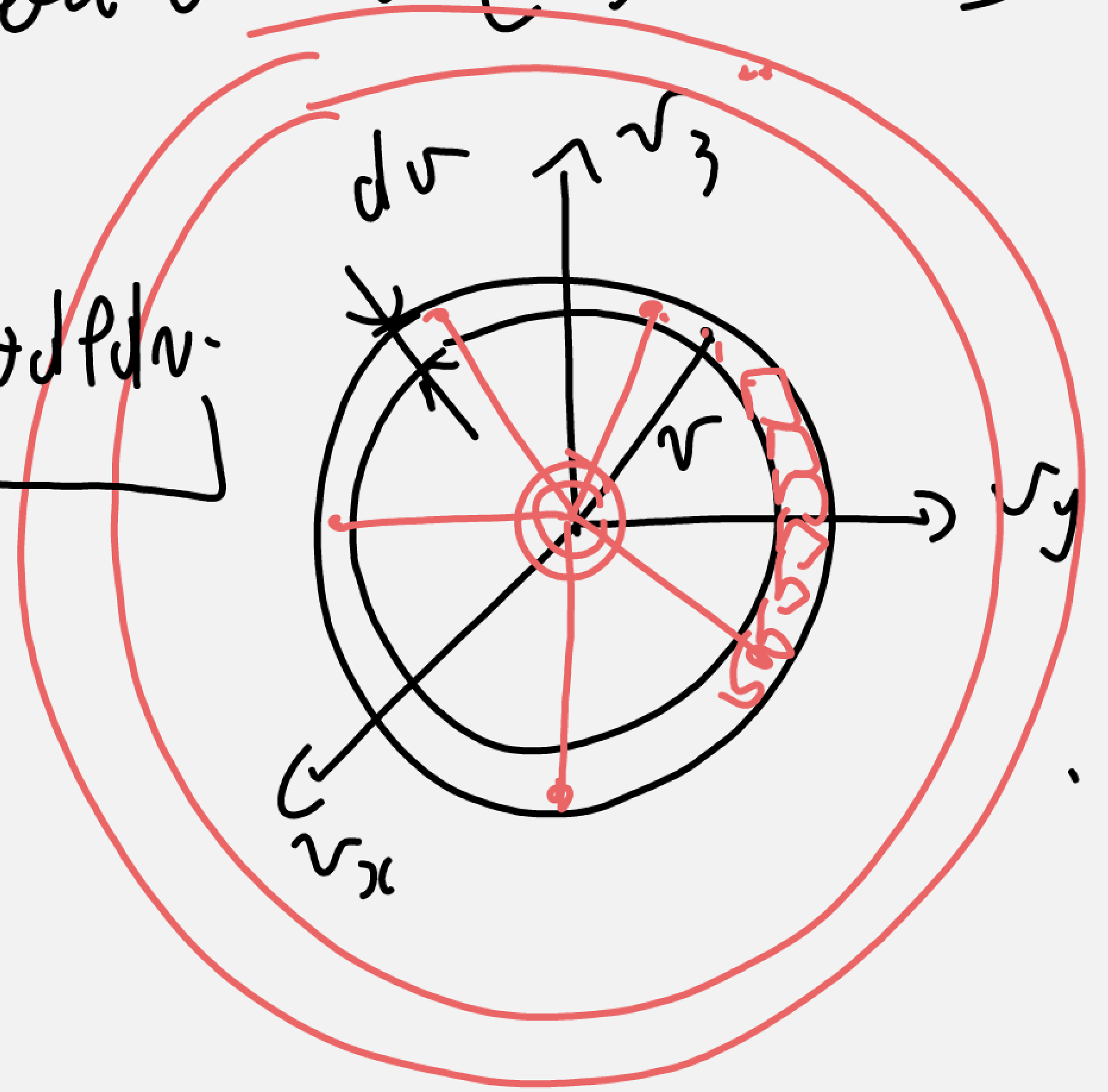


$$\rho(v_x, v_y, v_z)\, dv_x\, dv_y\, dv_z = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m v^2}{2 k_B T}\right) dv_x\, dv_y\, dv_z$$

$\rho(v)\,dv$ proba que la norme de la vitesse soit dans $[v, v+dv]$

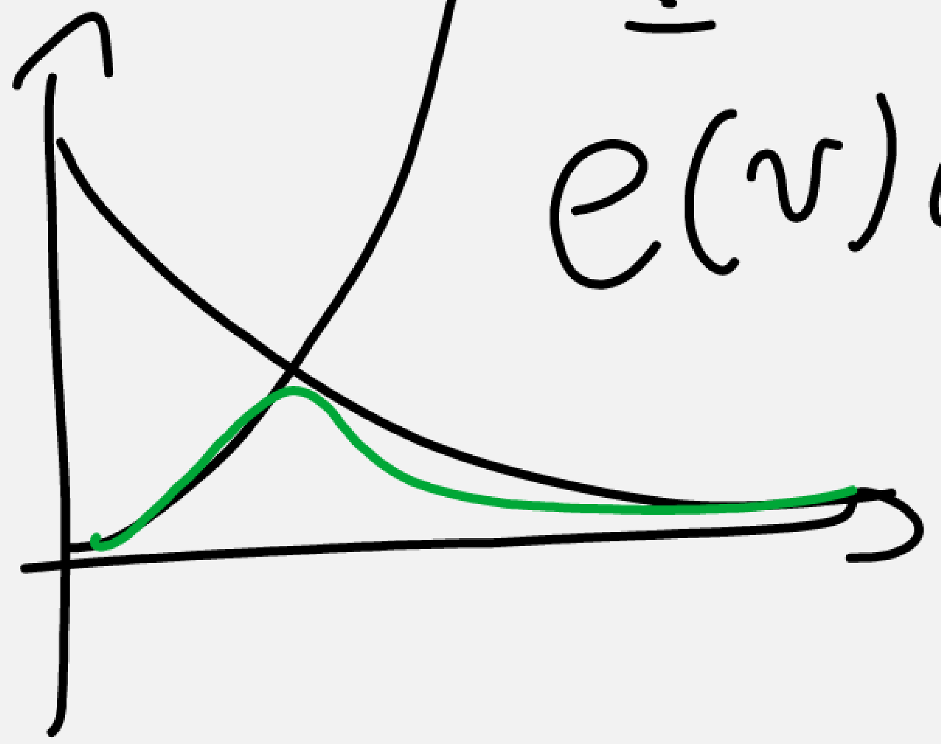
$$\rho(v, \theta, \varphi) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m v^2}{2 k_B T}\right) \times v^2 \sin\theta\, d\theta\, d\varphi\, dv$$

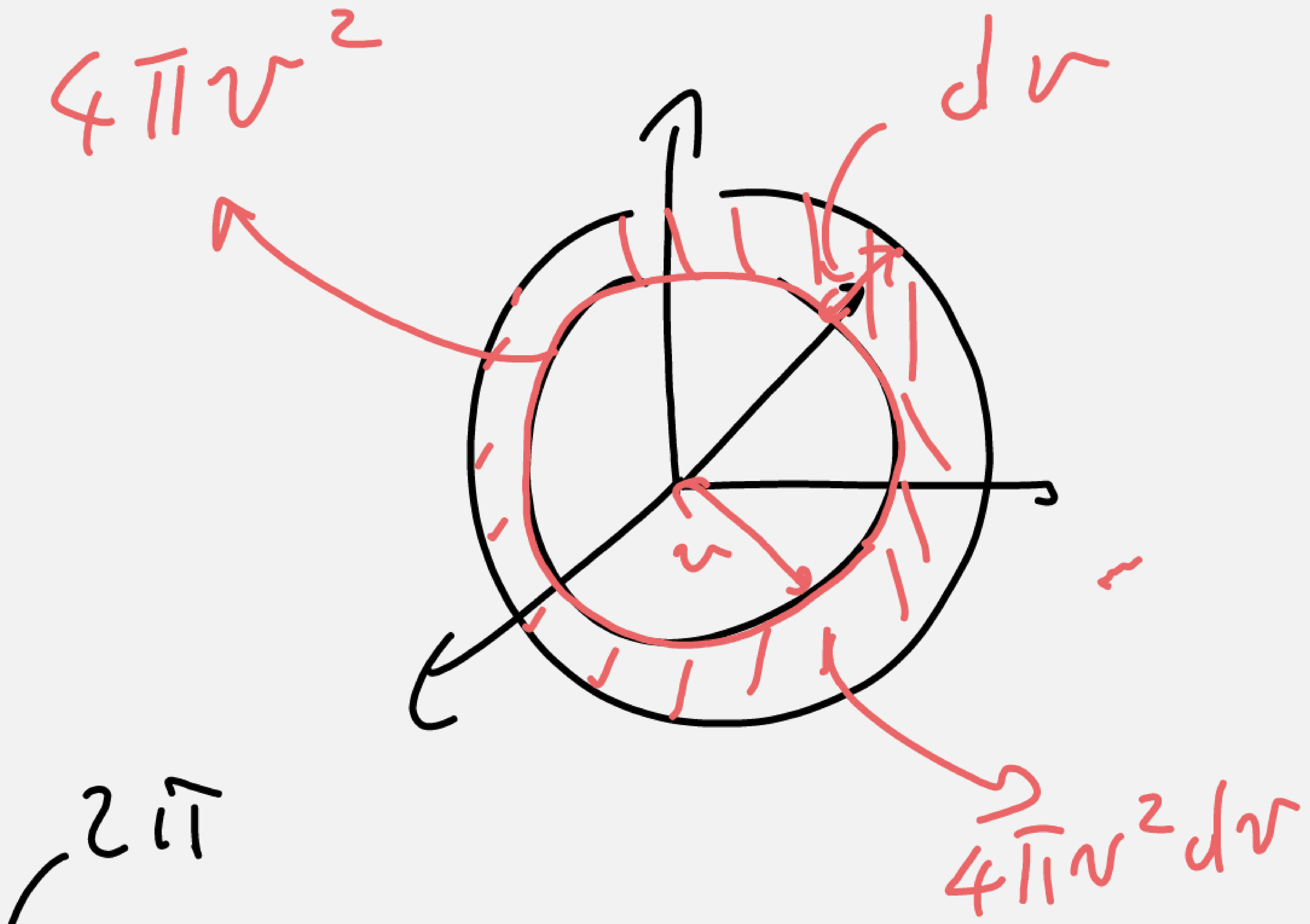
$\times\, d\theta\, d\varphi\, dv$



$$\rho(v,\theta,\varphi)\,dv\,d\theta\,d\varphi = v^2\sin\theta\left(\frac{m}{2\pi k_BT}\right)^{3/2}\exp\left(-\frac{mv^2}{2k_BT}\right)dv\,d\theta\,d\varphi$$

$V(v+dv)-V(v)$





$$\rho(v)\,dv = \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi}\rho(v,\theta,\varphi)\,dv\,d\theta\,d\varphi$$

$$\int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi}\sin\theta\,d\theta\,d\varphi$$

$$\underbrace{\int_0^{\pi}\sin\theta\,d\theta}_{2}\times\overbrace{\int_0^{2\pi}d\varphi}^{2\pi}$$

$$\rho(v)\,dv = \underline{4\pi v^2\,dv}\;\underline{\left(\frac{m}{2\pi k_BT}\right)^{3/2}\exp\left(-\frac{mv^2}{2k_BT}\right)}$$